*Introduction*

Timber production is inherently a risky business. Stumpage price fluctuates all the time, creating uncertainty for forestland owners and managers. Over the years, rotation age determination under uncertainty has remained one of the key topics in forest management and economic. To overcome the stumpage price uncertainty, over the last 40 years, real options analysis has been applied to address this problem. Over the last 12 years, three extensive reviews of such applications have been published by Hilderbrandt and Knoke (2011), Yousefpour et al. (2012) and Chaudhari et al. (2016). Readers interested the details of these applications are refer to these three reviews.

As one of the stochastic dynamic programming applications, Brazee and Mendelsohn (1988), hereafter B&M, devised the reservation price method to determine both the value of the land and trees as well as the rotation age. As Plantinga (1998) pointed out, the reservation price of B&M method represents a mechanism for incorporating the option value in determining the optimal rotation length**.**  Since the publication of B&M in 1988, applications of the stochastic dynamic programming techniques include Clarke and Reed (1989), Haight and Holmes (1991), Insley and Wirjanto (2010), Lu and Gong(2003),Mohammadi Limaei and Mohammadi (2021), Rakotoarison and Loisel (2017), Thorsen (1999), and Yin and Newman (1997). In addition, other highly sophisticated techniques such as Alvarez and Koskela (2007), Chladná (2007), Insley (2002), Morck et al. (1989), Sodal (2002) and Willassen (1998) have been employed to address forest management under price uncertainty. Still further, applications of numerical methods include articles such as Duku-Kaakyrie and Nanang 2004, Gjolberg and Guttormen (2002), Hughes (2000), Insley (2002), Insley and Rollins (2005), Khajuria et al. 2012, Manley and Niquidet 2010, Mei and Clutter (2013), Petrasek and Perez-Garcia (2010), Rocha et al. (2006), Thomson (1991, 1992) have been published.

All of them, however, tried to internalize the effects of price uncertainty. As a result, forestland owners remain as passive price takers in the market. When the market stumpage price is low, forestland owners have no choice but to wait.

In this study, we will outsource the price uncertainty with an American put option and apply the binominal tree method of Cox et al. (1979) to establish the option value at various stand ages and their reservation prices. Further, rolling put options of different lengths are employed to determine the reservation prices. By picking a specific strike price plus lengths for the put option, and, consequently, the resulting reservation prices at various ages, forestland owners become active price setters in the stumpage market. Rather than merely waiting for a good price in the future, they would leverage the financial tools to manage downside price uncertainty actively. Instead of passive forest management, this development would bring about a new era of active forest management for forestland owners.

Conceptually, a put option enables the forestland owners to have the choice but not the obligation of selling a specified amount of timber for a specific price (strike price) by a particular date. An American put option allows the owner of the put option to exercise it any time over its duration while the European put option can only be exercised at the end of the contract. The American put option is chosen in this study because such a purchase eliminates certain level of price uncertainty over that duration of the contract. More importantly, the derived option values enable the forest owners/managers to establish the reservation prices at various stand ages to take advantage of potentially high stumpage prices that might occur.

When the downside stumpage price uncertainty is covered with rolling American put options, the option values as well as the reservation prices at various stand ages can be established. When the spot market price of the stumpage exceeds that of the reservation price, the stand should be harvested for superior financial returns. In doing so, timber production becomes an active venture with forestland owners selecting the length of the rolling put option, the starting and ending year of the management window, and the strike price. As a result, forestland owners choose a specific set of reservation prices appropriate for their uncertainty tolerance and determine the corresponding rotation age plus land expectation value accordingly.

In its simplest form, the forestland owner out-sources such uncertainty with an American put option O(P1(T1), τ1) Q1(T1) at age τ1, τ1 < T1 with the specified strike price of P1(T1) for a stand volume Q1(T1), by the expiration age T1, T1 ≥t1. In this study, the put option would start at age τ1=15, for an expiration age of T1=70 while the harvest age t1 could be somewhere between 15 and 70. To provide enough time to realize the benefit of the put option, T1 is longer than twice the optimal rotation age under certainty. But since the longer the T1-τ1 is, the more expensive the put option would be. Further, the higher the specified strike price P1(T1) and the higher the stand volume Q1(T1), the costlier the put option would be. Therefore, judicious selections of these variables for the put option represent important management decisions in establishing the reservation prices.

*Simulations and Analyses*

In this section, the results of the reservation prices established with the American put option and that by B&M approach are determined and compared. Once the reservation prices are established at various stand ages, simulations are carried out to determine the average rotation age and LEV1 -- the land expectation value at the beginning of the first rotation under the generalized Faustmann formula (Chang 1998). For comparison with the results of B&M, the cost of purchasing the American put option is not considered.

(1)

Where

V1(t1) =P1(t1)Q1(t1) represents the stumpage value of a t1-year old stand per acre with P1(t1) being the stumpage price of the t1-year old stand and Q1(t1) being the stand volume per acre of the t1-year old stand.

A1,s represents the miscellaneous annual income or expenses per acre during the first rotation for year s, 0 < s < t1.

C1 represents the regeneration cost per acre of the first rotation.

r1 represents the annual interest rate during the first rotation.

LEV2 represents the land expectation value per acre at the end of the first rotation and the beginning of the second rotation. It embodies all the optimal rotation ages of future rotations and represents the present value of profits from all future rotations.

LEV1 is chosen in this study instead of the classical land expectation value (LEV) proposed by Faustmann (1849) because the stumpage price with all its uncertainty, stand volume, regeneration cost, and interest rates are unlikely to repeat themselves from rotation to rotation forever.

Given the relevant parameters shown in Table 1, the optimal rotation age under certainty will be 31 for loblolly pine and LEV1 of $1921.35/A. An expiration age of 70 is chosen to ensure that it is long enough to capture the benefits of price fluctuations. To calculate the put option value, the average real stumpage price of southern pine in Louisiana from 1956 to 2015 of $169.19/MBF with a standard deviation of $65.73/MBF (Zhang and Chang 2018) is chosen as both the spot and strike prices. As shown in Table 1 with an interest rate of 4%, the option value calculated by the binominal tree method of Cox et al. (1979) method, at age 15 the 55-year American put option is valued at $61.62 per MBF and gradually declines to $0 per MBF at age 70. For any year ω­1, ≤ ω­1 ≤ , the reservation price RP(ω­1) can be calculated as follows:

RP(ω­1) = + (2)

For example, given the settings above, the reservation price at age 15 would be

RP(15)=169.19+ 61.6171\*(83640.72/5236.11) = $1153.46/MBF

This reservation price at age 15 is so high because the option value of $61.62/MBF must be multiplied by 15.97 (83.64 MBF at age 70 divided by 5.24 MBF at age 15). If the spot price at age 15 is higher than this reservation price, literally with no upside potential and 100% downside risk, an immediate timber harvest would be justified. As the stand age increases, the difference between Q(ω­1) and Q(70) narrows. Over time, the option value decreases, and the reservation price declines. By age 70, with an option value of 0, the reservation price is the same as the target price of $169.19/MBF. Compared with the reservation prices obtained by the B&M method, Figure 1 shows that the reservation prices obtained by the 55-year put option approach are higher everywhere than those of B&M. The gap between the two is at its widest initially, then narrows down over time until they converge at age 70.

To examine the impact of the reservation prices of these two methods on the optimal rotation age and LEV1, 50,000 simulations each are carried out. Shown in Table 2 are the average rotation age and LEV1 of the two methods. The 55-year American put option method results in an average optimal rotation age of 46 years and an average LEV1 of$2633/A. The former is 50% longer than the 31 years under certainty and the latter is 37 % higher than the $1921/A under certainty. When compared with the results of the B&M method, however, the former is 10.7 years longer than the 35.45 years of the B&M method and the latter is $250/A less than the $2883/A of the B&M method. Visual inspection of the reservation price curves of Figure 1 indicates that the reservation price curve of the 55-year American put is tilted backward towards the end of 70 years. Not surprisingly, it results in the much longer average rotation age.

*Rolling put options*

Given that the full-length put option of 55 years creates very high reservation prices during its earlier years and results in longer average rotation age, rolling put of shorter durations are explored. Specifically, 10, 13, 15, 17, 19, 20, 21, 23, 25 and 40-year rolling options are explored. For a rolling 10-year put, it will start with a put option from age 15 and ends at age 25, to be followed by a put option from age 16 to age 26 and so on. By age 61, it will then conclude with the last 10 put options of 9, 8, 7, 6, 5, 4, 3, 2, 1 and 0 year. The reservation prices obtained with the rolling 10, and 20-year rolling put options are shown in Table 1 and Figure 1. Due to their shorter option durations, the reservation prices of the 10 and 20-year rolling puts tend to be lower and flatter, which, in turn, affect both the average rotation age and land expectation value.

Table 2 presents the average LEV1 and rotation age plus their standard deviations from 50,000 simulations for rolling put option of various lengths. Among them, the 15-year put rolling option results in the highest average LEV1 of $2872.06/A with an average rotation age to 35.69 years. These results are, essentially, the same as the average LEV1 of $2882.60/A and rotation age of 35.45 years obtained with the B&M method. Interestingly, the 10-year rolling American put option results in a flatter reservation price curve. The average rotation age shortens to a surprising 29.86 years and an average LEV1 of $2771.68/A. The former is even shorter than the rotation age of 31 years while the latter is 44% larger than the LEV1 of $1921.35/A under certainty. Furthermore, the LEV1 with the 10-year rolling put option is only 3.5% less than that of the 15-year rolling put option. As shown in Figure 2, the land expectation value peaks with the 15-rolling put option. Longer rolling put options afterwards result in steadily declining land expectation value, with that of the 40-year rolling put option within $6 of that of the 55-year option.

*The meaning of the rolling put options*

When a rolling put option of shorter duration is deployed, only a portion of the stand volume is covered by the put option when compared with the case of the 55-year long put option. For example, at age 15 the rolling 15-year put option at age 15 will cover 30,536 board feet of the stand volume at age 30. Compared with the 83641 board feet stand volume of age 70 covered by the 55-year put option, this translates into a partial coverage of 36.51. Similarly, the rolling put option at age 20 will result in a partial coverage of 46.97%. As shown in Table 3, as stand age increases, the coverage ratio gradually increases to 100% at age 55 and thereafter. Also presented in Table 3 is the coverage ratio of the 10-year and 20-year rolling put options. These coverage ratios lead to two observations. First, rolling put options result in partial coverage which increase with stand age, eventually reaching 100%. Second, the shorter the duration of the rolling put option, the lower the coverage ratio. By choosing a shorter rolling put option, the forestland owners are expressing their tolerance of uncertainty – the shorter the put option, the more uncertainty tolerant. Borrowing from Arrow’s (1971) terminology on risk bearing, shorter the rolling put options, more risk seeking.

*Discussion*

Regardless of the duration of the put options, when compared with the outcome under certainty assumption, outsourcing stumpage price uncertainty with a put option increases the timberland value expressed as LEV1. Moreover, Figure 2 also shows that the duration of the rolling put option and the average LEV1 exhibits a curvilinear relation. Since buying a full length 55-year put option means the forest landowner is extremely uncertainty (risk) averse, wanting to cover all downside stumpage price uncertainty. A put option with a shorter duration, on the other hand, might provide a choice for people who are more tolerant of uncertainty. In other words, the length of put option reflects forestland owner’s level of uncertainty tolerance. The curvilinear relation between the land expectation value and the length of the rolling put option indicates that a specific level of uncertainty tolerance exists to maximize LEV1,reaffirming the finding of Zhang and Chang (2018).

Even though the 15-year rolling put option produces the best financial outcome, note that between a rolling option duration of 13 and 20, their land expectation values are within $45/A or 1.6% of that of the 15 years, suggesting that the length of the rolling put option makes a relatively small difference within the neighborhood of the optimal rolling put option length. Further, the surprisingly small difference in LEV1 between a rolling 10-year and 15-year put options of only $101.27/A ($2872.95-2771.68) or 3.5% seems to suggest that for the current example being too tolerant of uncertainty, while not advisable, may only carry a small price. On the other hand, forestland owners may want to shun away from choosing longer rolling puts, which results in significantly lower land expectation values. Surely, additional research is required to confirm if these initial conclusions remain valid for stumpage prices with wider standard deviation or different species. Furthermore, as shown in Table 4, for the 15-year rolling put option, .7% of the time timber harvests occur before age 20; 0.78% of the time occur they occur between 60 and 65 years, and .33% of the time they occur after age 65 for a total 1.81% of the time. Thus, the financial impact of selecting the starting and ending ages of the management window represents a question of high practical relevance. Lastly, field forestry operations often involve annual income from hunting lease, carbon sequestration, and other payment for ecosystem services as well as annual management expenses. How such inclusions would affect the option value and consequently reservation price, harvest age and the financial outcome of management decisions awaits careful exploration.

*Conclusions*

This article demonstrates that the American put option can be used effectively to outsource stumpage price uncertainty and enhance the land expectation value. The reservation prices derived from the American put options for various stand ages establish boundary values for forestland owners to determine whether the market price is high enough to justify an immediate timber harvest. With this approach, the forestland owners could realize higher harvest revenue and higher land expectation values to the tune of 40%. This efficacy in enhancing the land expectation value is beyond disputes. The actual figures of the land expectation value reported here are of lesser importance because they depend critically on the interest rate and other input factors used in the calculation.

With the American put option to determine the option value and reservation price, we are ushering in the era of active forest management which allows timberland owners to actively manage stumpage price uncertainty in timber production. They can also explore different combinations of strike price, interest rate, the length of the management window, and the length of the rolling put options for the desirable reservation prices, resulting in specific land expectation value and rotation length.

Once forestland owners begin actively selecting those relevant parameters, they are no longer price takers. Instead, they become price setters. That would cause a sea change in the stumpage market. As Gong and Löfgren (2007) have shown, its implications on timber supply and social welfare could be profound and need to be fully explored.

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